



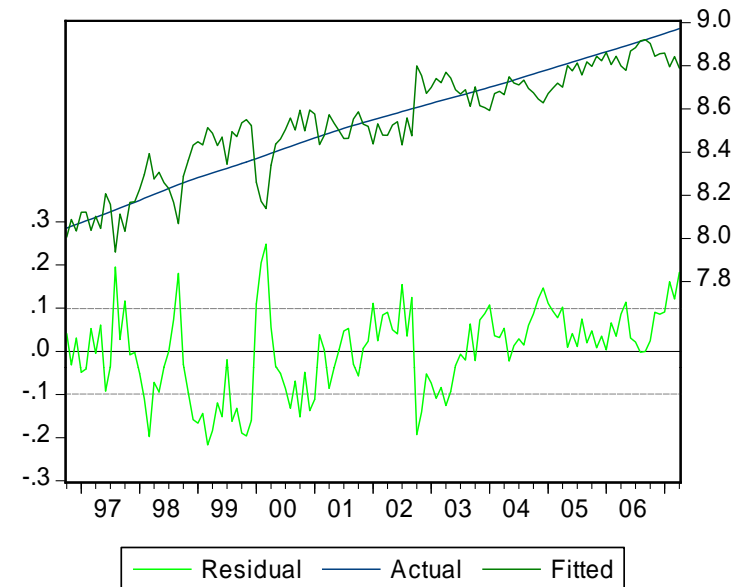
Cointegrating portfolio
CAC-Dax-SMI
April 2007

Our cointegrating method

- Aim is to find a minimum-variance portfolio that delivers a 'benchmark plus' return over time
- Step one: construct the benchmark index
 - Here we use Libor + 4%
- Step two: look for combinations of assets that 'cointegrate' with that benchmark
 - Weighted combination of assets that deliver Libor + 4% on average
 - Weights come from cointegrating regressions
 - Test residuals to check that they mean revert over a reasonable period of time
- Step three: back test portfolio performance out-of-sample
 - Look at average returns out of sample (a test of equation stability)
 - Assess daily risk out of sample

A cointegrating portfolio

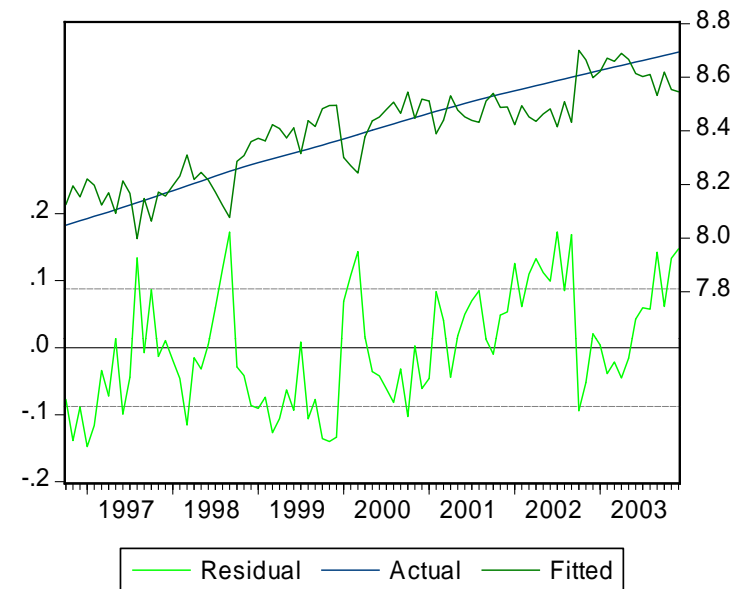
Dependent Variable: LOG(TBILLPLUS4_IND)				
Method: Least Squares				
Date: 04/26/07 Time: 15:27				
Sample (adjusted): 1996M10 2007M04				
Included observations: 127 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.730722	0.410277	9.093174	0
LOG(FRA)	1.394916	0.076523	18.22865	0
LOG(GER)	-2.039468	0.080524	-25.32742	0
LOG(SWI)	1.123903	0.108429	10.36537	0
R-squared	0.86285	Mean dependent var		8.531562
Adjusted R-squared	0.859505	S.D. dependent var		0.26342
S.E. of regression	0.098737	Akaike info criterion		-1.761727
Sum squared resid	1.199123	Schwarz criterion		-1.672147
Log likelihood	115.8697	F-statistic		257.9435
Durbin-Watson stat	0.673782	Prob(F-statistic)		0



- This is an example of a cointegrating portfolio – it has delivered Libor + 4% on average
- Equation is estimated (full sample) from end-1996 through to April 2007
- Regression suggests weights of 139% on the CAC-40, -204% (short) the Dax, and 112% on the Swiss SMI (all indices are expressed in local currencies, so this is an unhedged portfolio)
- These weights are approximately 100%, -200% and 100% - in other words a fully invested long-short on the CAC v Dax, coupled with another fully-invested long-short on the SMI v Dax
- The residuals pass two tests of cointegration – the augmented Dickey-Fuller test and the Johansen test (at 95% confidence level)
- Also – and this is key – the residual is currently strongly positive and outside its one-standard-error band.
- This implies that portfolio performance should be even greater than Libor + 4% over the next few months, so long as the cointegrating relationship holds

Estimating to end-2003

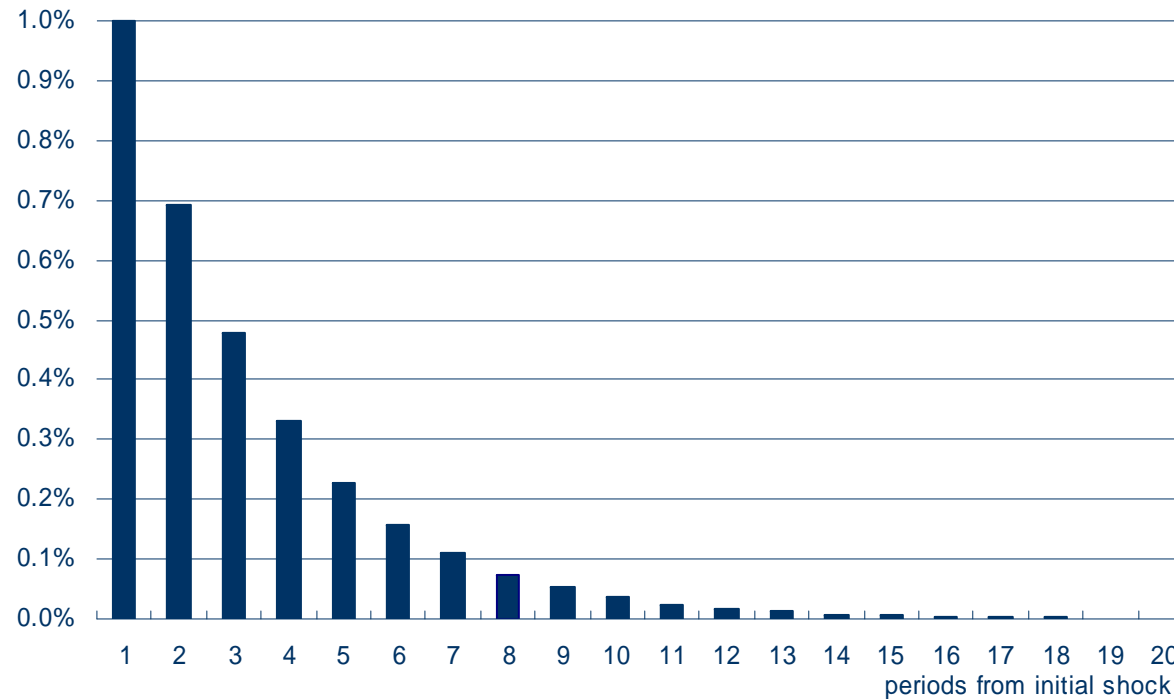
Dependent Variable: LOG(TBILLPLUS4_IND)				
Method: Least Squares				
Date: 04/25/07 Time: 15:42				
Sample (adjusted): 1996M10 2003M12				
Included observations: 87 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.152225	0.525376	11.71013	0
LOG(FRA)	1.227828	0.073612	16.67968	0
LOG(GER)	-1.467941	0.105176	-13.95701	0
LOG(SWI)	0.46647	0.136148	3.426186	0.001
R-squared	0.796732	Mean dependent var		8.392393
Adjusted R-squared	0.789385	S.D. dependent var		0.191002
S.E. of regression	0.087656	Akaike info criterion		-1.985901
Sum squared resid	0.63774	Schwarz criterion		-1.872526
Log likelihood	90.38669	F-statistic		108.4424
Durbin-Watson stat	0.750796	Prob(F-statistic)		0



- Having estimated over the full sample, we re-estimate over a shorter sample, leaving more than three years of out-of-sample data on which to back test
- The coefficients change a bit over the smaller sample – now 122%, -147% and 47% on CAC, Dax and SMI respectively
- These coefficients are similar to $100\% \cdot (\text{CAC-DAX}) + 47\% \cdot (\text{SMI-DAX})$ – i.e. a fully invested long-short in the CAC v Dax plus a 47% invested long-short in SMI v Dax
- So some signs of instability, but basic principle of a portfolio involving two long-short positions still holds
- Also, the residual was strongly positive at the end of 2003 – just as it is now

Mean reversion properties

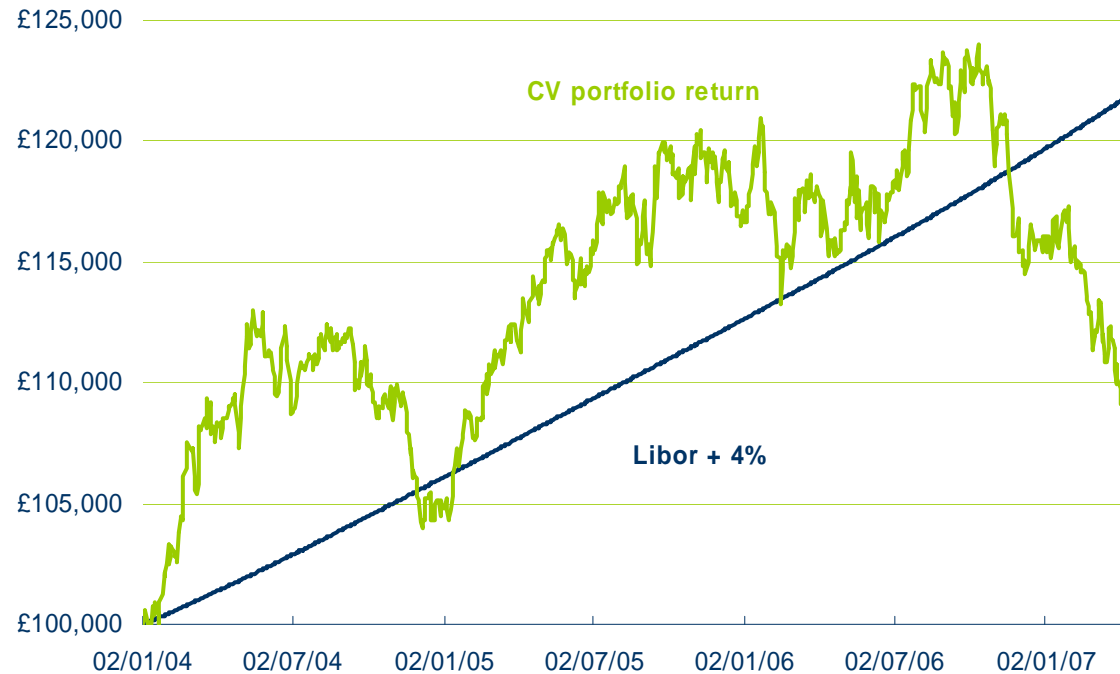
Dynamic properties of residual



- A shock of 1% to the residual has almost died out within 12 months, and is fully eliminated within 18 months
- That suggests the sort of horizon we should be looking at for investment purposes is around the same

Out-of-sample performance

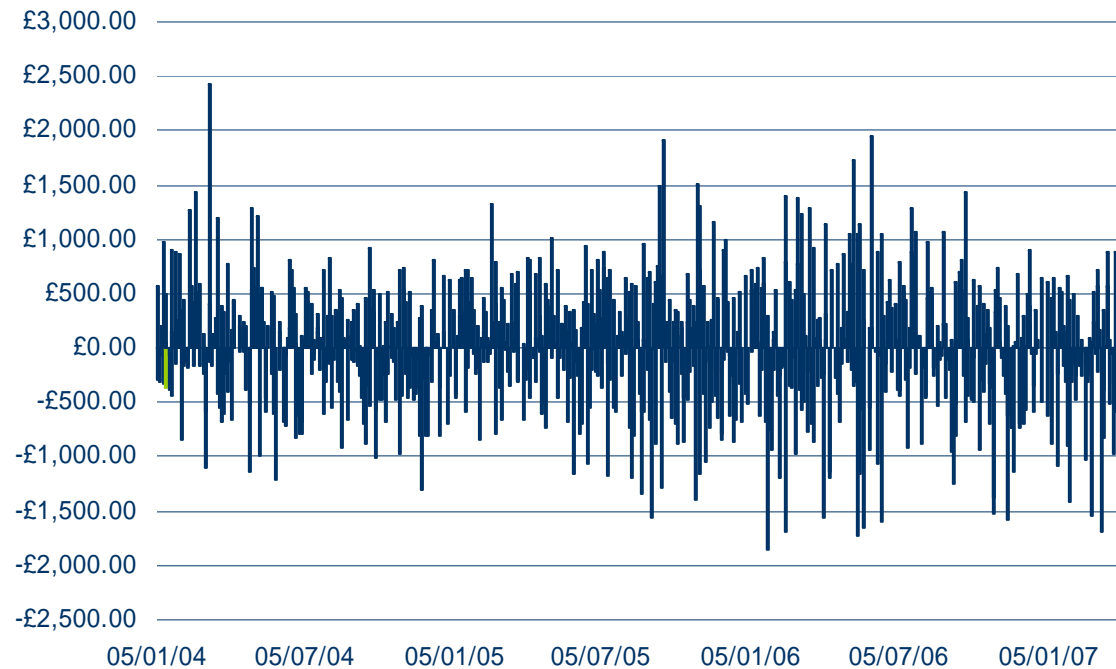
How the estimated CV tracks Libor +4% out of sample



- This chart shows how the portfolio – using weights from equation estimated to end-2003 – has performed out of sample
- In back testing – though not the estimation – we mark to market every day during the out of sample period
- We implement the portfolio as a per point bet (this is similar to using futures contracts) and we do not deduct trading costs (not a big issue since the portfolio is not rebalanced during this period)
- The initial fund size is £100,000 and the green line shows how this fund value changes over time
- The blue line shows the performance of the Libor+4% benchmark
- For much of the out-of-sample period the portfolio outperforms Libor + 4%, but it has had a poor run of late (after more than three years of daily marking to market)

Daily P&L out-of-sample

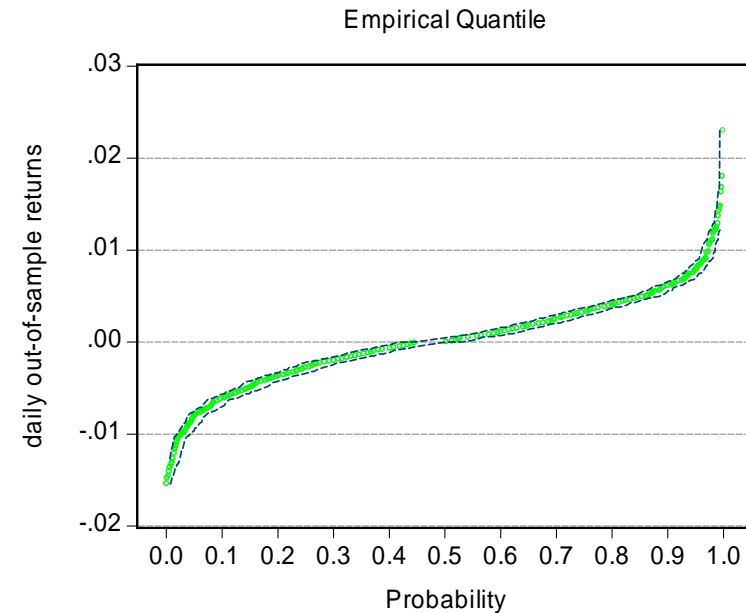
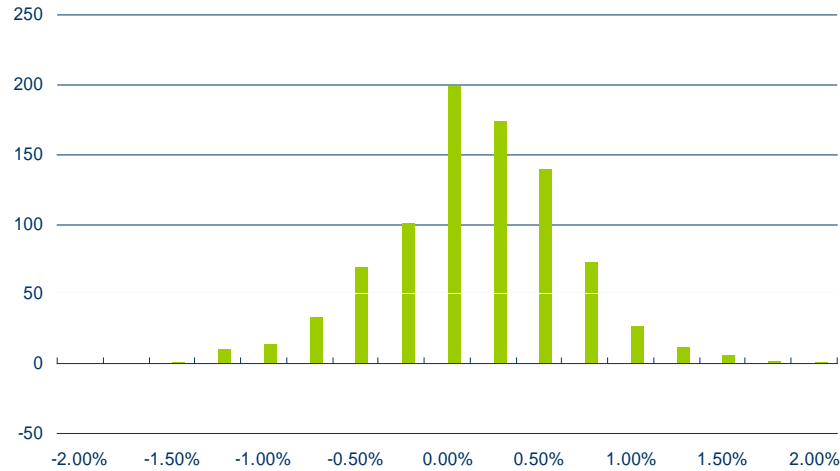
Daily P&L - estimated CV coefficient weights



- We also track the daily P & L implied by the portfolio out of sample
- The daily variance is not exceptionally high, though stop-loss limits would have to have exceeded -2%

Distribution of daily returns

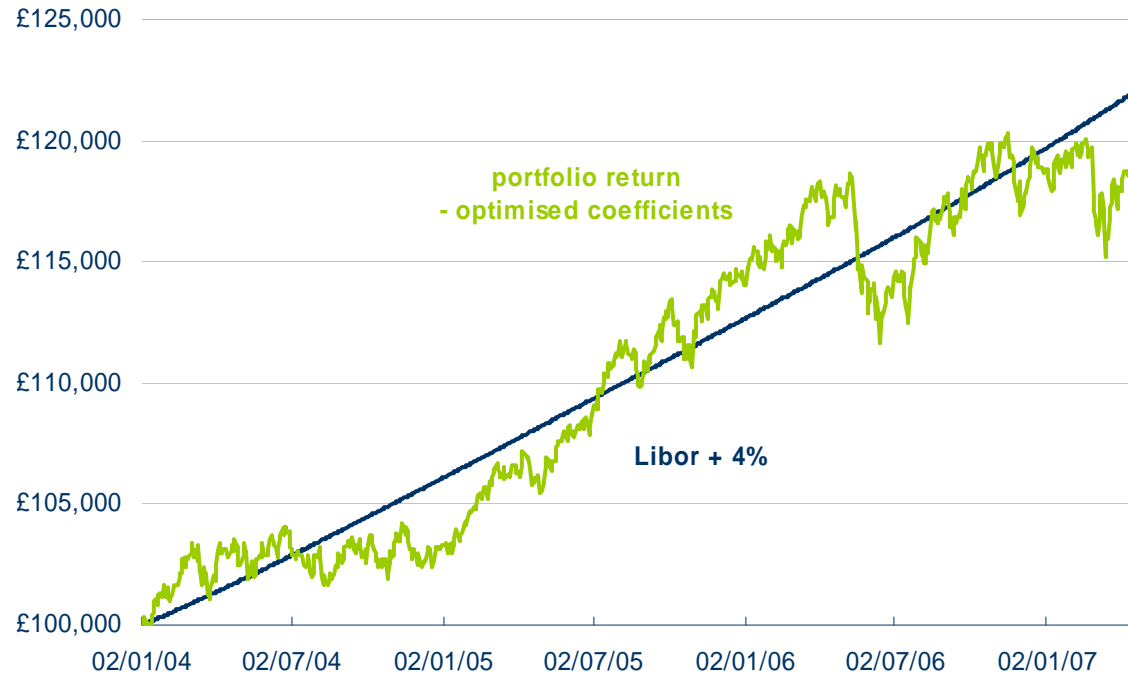
Distribution of daily returns - estimated CV weights



- These are the same daily P & L data shown as a distribution
- The left-hand chart suggests there is a small up side skew to daily returns
- The right-hand chart is an empirical quantile chart
 - The horizontal axis shows the probability that returns are less than a given number
 - The vertical axis shows what that number is
 - So there is around a 20% chance that daily returns are less than -1/2%
- Somewhat surprisingly the portfolio has not experienced extreme losses in the period
- The 1% 1-day VaR is -1.33%
- In reality, a cautious approach to risk would assume something significantly larger than that

Optimising portfolio weights

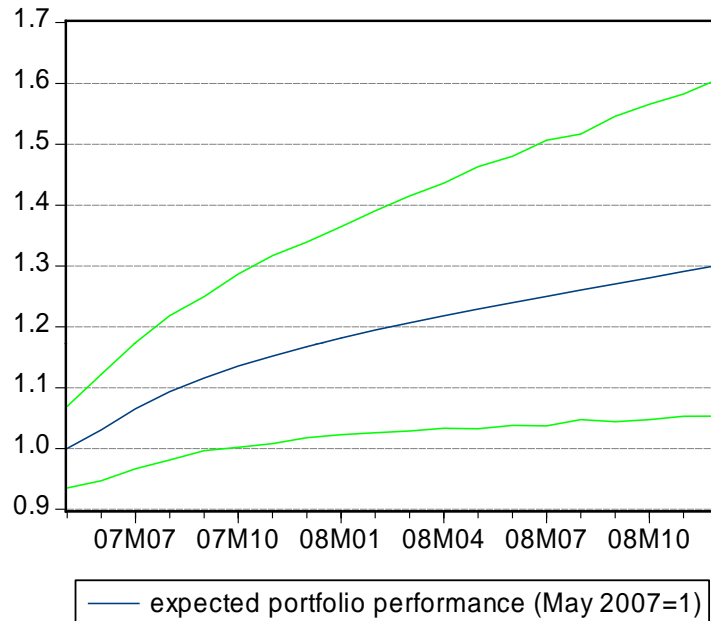
Optimal weights for tracking Libor + 4% out of sample



- We can 'cheat' and optimize the portfolio weights to minimise the tracking error around Libor + 4% out of sample.
- Doing that changes the portfolio weights again significantly – now 50%, -29%, and 17%
- Although the weights change, the sign and relationship between the size of the coefficients remains broadly the same
 - we are still talking about something like a combination of two long-short portfolios

Simulating the portfolio

Our portfolio +/- 1 sigma



1y expected portfolio performance

Expected return	22.9%
Portfolio standard error	23.4%
Sharpe ratio	0.98

- Going back to the full-sample estimates, we can do a stochastic (Monte Carlo) simulation to generate forecasts of the portfolio's performance from now on
- To do this we generate thousands of outcomes for the CAC, Dax and SMI, based on their past performance and inter-relationships – mean returns, variance and covariances
- The chart shows how the weighted combination of these outcomes would be expected to perform (the blue line), together with one standard deviation around this outcome
- Portfolio performance should remain between the green lines two-thirds of the time

Conclusions

- In this note we have used cointegration analysis to identify a portfolio that has delivered Libor + 4% on average over 10 years
- The portfolio we identify involves (approximately) two long-short positions – one in the CAC v Dax, the other in the Swiss SMI v Dax (all expressed in local currencies)
- In out-of-sample daily back testing we find that the portfolio has reasonable-looking risk properties
- And simulating forward using Monte Carlo analysis it has a reasonable risk-return trade-off

- A disclaimer: of course, this note is meant as idea generation and is not an invitation to implement exactly the portfolios presented without further in-house testing